Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

of 1876 he left the academy at Cheshire to become a tutor in mathematics at Yale. He continued his studies in mathematics and in other departments, and his advancement to the Doctor's degree and to higher appointments was steady and natural. Anyone who never knew him might ask how a man who never was an undergraduate student for a day in a college, or even a pupil in a high school, should become the head of the Yale Graduate School.

Professor Phillips had much technical mathematical ability and wrote several books and numerous articles for the mathematical journals. It is a significant fact that each one of his books was written in collaboration with one of his colleagues. Dean Henry P. Wright said of him: "He lived to do good and make others happy, and could do nothing to lessen the happiness even of those who had injured him. He was wholly unselfish. I cannot think of him as forming any plan solely for his own advancement. He seemed to ignore his own interests in his devotion to the interests of others. He was always busily occupied, but it was for other persons or for other objects than his own—for the Cheshire school, or the Hopkins grammar school, or the Hotchkiss school; for the college; for the graduate department; for the Bicentennial Fund; for St. Thomas's church in New Haven; for his students, and those who had been his students; for his colleagues; for his friends, or his family; for the assistants in his office, or for the servants in his house. He was always ready to help when an opportunity offered."

As a teacher of young men, he was firm and kind. Every student who came in contact with Andy, as we used to call him, immediately felt his wonderful personality. His love and enthusiasm for work were so great that even the laziest student somehow managed to work some for him. One of them once said: "Work is catching in Andy's class." His interest in his students was so great that every student who took any course with him felt that he was an intimate and personal friend. His memory is now dear to all his students, not for the mathematics he taught so well, but for the high ideals he inculcated by his beautiful character and example. His students and friends will always think of him as they knew him, full of life, sunshine, and human sympathy.

H. T. Burgess.

University of Wisconsin.

BOOK REVIEWS.

Send all communications to W. H. Bussey, University of Minnesota.

Diophantine Analysis. By Robert D. Carmichael. John Wiley and Sons, Inc., New York, 1915. vi+118 pages. $1.25.

Greek theory of numbers, like Greek geometry, has come down to us in a remarkably disconnected and unsystematized form. The method used in solving one problem gives little hint of the method to be used in solving a second which may be apparently closely related to the first. The theorems in Euclid are arranged in masterly sequence, but the proofs stand by themselves, a different scheme for each. In Diophantine analysis, not only do the solutions stand apart from each other, but the theorems themselves seem unrelated and patchy. This characteristic has made it the favorite hunting ground for the amateur, who finds there little necessity for relating his work to other theories with which he may not be on speaking terms. This same characteristic has also attracted to the subject some of the finest minds in the world of science: men who have labored to pick out of this vast, confused heap of unrelated materials some connecting thread with which to bind the whole theory together. No one thing indicates more clearly the miraculous insight of Fermat than his instinctive appreciation of the importance of the so-called Pellian equation. Whether he had the key to his famous last theorem or not, he certainly had some clear notion of the place it should occupy in the Theory of Numbers. Fermat tried to do for Diophantine Analysis what his great contemporary, Desargues, did for pure geometry. Since Fermat's discovery of the method of "infinite descent" very few general methods have been found applicable to this kind of analysis. Since all solutions of the Pellian equation seem to be only disguises of the solution by continued fractions, it is not improbable that Fermat was familiar with this powerful algorithm.

Professor Carmichael in his book calls attention to two other methods which he calls the "method of multiplicative domains," and the "method of functional equations." As an illustration of what he means by the "method of multiplicative domains" he derives from the identity:

$$(m^2 + n^2)^3 = (m^3 - 3mn^2)^2 + (3m^2n - n^3)^2$$

the following two double-parameter solutions of the equation,

$$x^2 + y^2 = z^3:$$

$$x = m^3 + mn^2, \qquad y = m^2n + n^3, \qquad z = m^2 + n^2;$$

$$x = m^3 - 3mn^2, \qquad y = 3m^2n - n^3, \qquad z = m^2 + n^2.$$

The theory of the "multiplicative domain" is developed at some length in Chapter II. The above illustration indicates what may be deduced from the fact that the sum of two squares multiplied by the sum of two squares is again the sum of two squares. This general multiplicative property does not hold for the sum of two cubes, and Professor Carmichael shows how by increasing the number of variables a new set of numbers may be obtained which includes the given set and enjoys the multiplicative property. Thus, instead of the numbers represented by $x^3 + y^3$, which do not form a multiplicative domain, he uses the numbers $x^3 + y^3 + z^3 - 3xyz$, which do form such a domain. The underlying theory is contained in the theory of composition of forms. Thus the theorem that "No form can be transformed into the product of two forms of the same sort (irreducible forms being understood to have at least one invariant different from zero), unless the number of its indeterminates is a multiple of its order," indicates why the numbers $x^3 + y^3$ do not form a multiplicative domain.

In the "method of functional equations" Professor Carmichael makes use of rational solutions of certain types of Diophantine problems of importance in the work of Diophantus itself. A general systematic development of this method is not attempted.

Chapters III and IV, on the equations of the third and fourth degrees, are very valuable in giving in elegant form the principal results so far obtained in this field. Professor Carmichael has done good service in collecting and arranging not only his own researches but the results of the labor of others as well. A short account is given in Chapter V of certain higher equations, ending with a very complete statement of our knowledge regarding Fermat's Last Theorem.

Professor Carmichael has omitted entirely the theory of binary quadratic forms and has made no use of the theory of continued fractions. This has resulted in some rather unsightly patches in the book. The discussion of the equation $x^2 - Dy^2 = z^2$ makes the author almost as much trouble, and takes up almost as much space as a straightforward discussion of the continued fraction representing a quadratic surd would require.

The book will serve a valuable end in stimulating interest in this delightful branch of mathematics. There are many interesting and suggestive problems at the end of each chapter, some of which are excellent material for further research.

D. N. LEHMER.

UNIVERSITY OF CALIFORNIA.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

[Send all Communications to B. F. FINKEL, Springfield, Mo.]

PROBLEMS FOR SOLUTION.

ALGEBRA.

457. Proposed by FRANK IRWIN, University of California.

If a be any number prime to m and m/a be developed as a continued fraction,

$$\frac{m}{a} = a_1 + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_{k-1}} + \frac{1}{a_k},$$

with $a_1 \neq 0$, then there will exist a number b such that $m/b = a_k + 1/a_{k-1} + \cdots + 1/a_2 + 1/a_1$. Show that $ab \equiv \pm 1 \pmod{m}$ and determine the sign.

458. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that n terms of the series $1 + 3 + 4 + 6 + 7 + 9 + 10 + \cdots$ is $\frac{1}{4}(n + 1)(3n - 1)$ when n is odd, and $n/2[(3n/2) + 1]$ when n is even.

459. Proposed by C. N. SCHMALL, New York City.

By d'Alembert's test, or otherwise, show that in the infinite series

$$x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \cdots + \frac{n^nx^n}{n!} + \cdots$$

the upper limit of the interval of convergence is $1/e$ where e is the Naperian base, *i. e.*, $x < 1/e$ when the series is convergent. (Bromwich's *Infinite Series*, pp. 28, 33.)